GEOMETRY.

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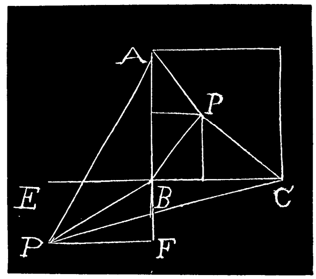
151. Proposed by FRANK A. GRIFFIN, Assistant in Mathematics, University of Colorado, Boulder, Col.

A point is at a distance of 1 inch, 2 inches, and $2\frac{1}{2}$ inches, respectively, from three corners of a square. Construct the square. Also solve for the general distances a, b, c.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; P. S. BERG, B. Sc., Larimore, N. D.; G. W. DROKE, State University, Lafayette, Ark.; and MARTIN SPINKS, Wilmington, Ohio.

Let P be the point, $PA=a$, $PB=b$, $PC=c$, $AB=x=$side of square, $PE=m$, $PF=n$, $m^2+n^2=b^2 \ldots.(1)$.



$a^2-(x+m)^2=a^2-x^2-2mx-m^2=n^2 \ldots.(2)$.

$c^2-(x+n)^2=c^2-x^2-2nx-n^2=m^2 \ldots.(3)$.

Adding (2) and (3) and using (1) we get

$$n+m=\frac{a^2+c^2-2b^2-2x^2}{2x} \ldots.(4).$$

Subtracting (3) from (2) we get

$$n-m=\frac{c^2-a^2}{2x} \ldots (5). \quad \therefore n=\frac{c^2-b^2-x^2}{2x}, \quad m=\frac{a^2-b^2-x^2}{2x}.$$

These values of m and n in (1) give

$$x^4-(a^2+c^2)x^2=b^2(a^2+c^2)-\frac{a^4+c^4+2b^4}{2}.$$

$$\therefore x=\{\tfrac{1}{2}[a^2+c^2\pm\sqrt{4b^2(a^2+c^2-b^2)-(a^2-c^2)^2}]\}^{\frac{1}{2}}$$

$=1.5163$ inches when P is without the square.

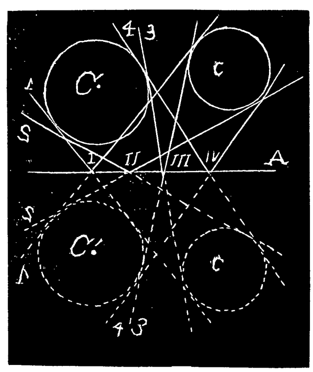
$=2.8197$ inches when P is within the square.

I. When P is without, lay off $PE=m$, perpendicular to PE lay off $EB=n$ and $BC=x$, also AB perpendicular to $EB=x$. The square is now determined.

II. Similarly, when P is within, only $EC=x-n$.

152. Proposed by ELMER SCHUYLER, Reading, Pa.

Find a point in a given straight line such than tangents drawn from it to two given circles shall make equal angles with the line.



Solution by MARCUS BAKER, Washington, D. C.

In the annexed figure let A be the given line, and C and c the given circles.

Regarding line A as an axis, revolve C and c about it until they fall at C' and c'.

Then draw the four common tangents 1, 2, 3, and 4 of circles C and c' intersecting the axis in I, II, III, and IV. Each of these points fulfils the conditions The proof is obvious.

Also solved by *G. B. M. ZERR*, and *D. B. NORTHROP*.